DOO-003-026203 Seat No. ________

M. Phil. (Science) (Sem. II) (CBCS) Examination

May / June – 2015

Maths : EMT-20011 : Complex Analysis

Faculty Code : 003
Subject Code : 026203

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : (1) Answer All Questions.
(2) Each Question Carries 14 Marks.,

1 Answer any seven questions. Choose the correct answer. 2×7=14

(1) If $z_{1,} z_2, \ldots, z_n$ are the only zeros of an entire function f repeated according to the multiplicity then the rank of f is ________

(A) n (B) ∞

(C) 0 (D) 1

(2) If f is an entire function of rank 2 and

$$f(z) = z\, e^{4z^2 + 2z^2 + 3z} \prod_{n=1}^{\infty} E_2\left(\frac{z}{n}\right)$$

then the genus of f is ________

(A) 2 (B) 3

(C) 4 (D) 1

(3) If f is analytic and injective on $\not\subset$ then f is ________

(A) a none-constant polynomial

(B) a rational function

(C) a transcendental entire function

(D) $f(z) = az + b,\ \forall\, z \notin \not\subset$ for some $a, b \notin \not\subset$ and $a \neq 0$

(4) _________ has infinitely many fixed points

(A) every transcendental entire function

(B) every entire function of finite non-integral order

(C) every entire function of finite order

(D) every non-constant polynomial

(5) _________ is locally path connected.

(A) every connected space

(B) every locally compact space

(C) every Riemann surface

(D) every path connected space.

(6) $M(r, \cos z) =$ ________, $\forall\, r \geq 0$

(A) $\cos r$ (B) $\cos hr$

(C) $\sin r$ (D) $\sin hr$

(7) $\int_0^{\pi} \log \sin t \, dt =$ ___________

(A) $\pi \log 2$ (B) $-\pi \log 2$

(C) 0 (D) $\frac{\pi}{\log 2}$

(8) If $f(z) = z^2 + 2z + 3$ then $f(B(0,3))$ contains a disc of radius __________

(A) $\frac{1}{72}$ (B) $\frac{1}{12}$

(C) $\frac{1}{36}$ (D) $\frac{1}{24}$

(9) ________ is a true statement.

(A) every analytic function from a compact Riemann surface to $\mathbb{C}$ is α constant function

(B) an entire function of finite genus μ is an entire function of finite order $\leq \mu$

(C) Weinstrass factorization of an entire function is unique

(D) order of $z^3 e^{z^2}$ is 3

(10) _________ is not a true statement

(A) every entire function of finite order λ is an entire function of finite genus $\leq \lambda$

(B) an entire injective function is a polynomial of degree 1

(C) ∞ is an essential singularity of every entire function

(D) $(\mathbb{C}, \exp)$ is a covering space of $\mathbb{C}^*$

2 Answer any two questions. 14

(a) State and prove Poisson-Jensen's formula.

(b) Define the order of an entire function. Give an example of an entire function of order 1 with proof.

(c) Let f be an entire function, $f(0) = 0$ and $n(r)$ be the number of zeros in $B(0, r)$ counting the multiplicity. Give an estimation for $n(r)$ and prove your answer.

3 (a) State, without proof, Hadamord's factorization theorem. State and prove the special case of Picard's theorem. 7

(b) If f is a transcendental entire function and p is a polynomial then prove that $\lim_{r\to\infty} \frac{M(r,\, p)}{M(r,\, f)} = 0.$ 7

OR

(c) If g is a polynomial of degree $n \geq 1$ then prove that order $\left(e^{g(z)}\right) = n$. 7

(d) State and prove the formula to find the order of an entire function. 7

4 Answer any two. 2×7=14

(a) If $D = \{z \in \mathbb{C} \mid |z| < 1\}$, $f = D \to \mathbb{C}$ is analytic, $f(0) = 0$, $f^1(0) = 1$ and $|f(z)| \leq M$, $\forall z \in D$ then prove that $f(D) \supset B\left(0, \frac{1}{6M}\right)$

(b) Define Landau's constant L. If $R > 0$ and f is analytic in a region containing $\overline{B}(0, R)$ then prove that $f(B(0, R))$ contains a disc of radius $R|f^1(0)|L$

(c) State and prove Little Picard's theorem.

5 Answer any two : 14

(a) Prove that $\mathbb{C}_\infty$ is a Riemann surface.

(b) Define analytic function between two Riemann surface. Prove that every polynomial "P" is in analytic function $P : \mathbb{C}_\infty \to \mathbb{C}_\infty$.

(c) Define isomorphism between two Riemann surfaces x, y. If $f : x \to y$ is an isomorphism of Riemann surface then prove that $f^{-1} : y \to x$ is also an isomorphism of Riemann surfaces.

(d) Define branch point of a non-constant analytic function $P : x \to y$ between two Riemann surfaces x, y. State and prove the condition under which $P : x \to y$ is unbranched.